Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Mathematical Physics and Classical Mechanics
Subject Code : 4SC05MPC1 **Branch: B.Sc. (Physics)**
Semester : 5 **Date : 26/02/2020** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Define : Periodic function (01)
b) What are the Dirichlet Conditions? (01)
c) Define Fourier series? Write its general formula and identify the coefficients. (01)
d) What kind of Fourier series are obtained for even and odd functions? (01)
e) What are the applications of the Fourier series in the field of physical sciences? (01)
f) Obtain Fourier coefficient a_0 for $f(x) = x$ in the interval $(0, \pi)$. (01)
g) Write the formula of a_0 , a_n and b_n for the extended intervals of $(-\ell, \ell)$ in the Fourier series. (01)
h) What is constraint and constrained forces? (01)
i) Name different types of constraints. (01)
j) Define : Generalised coordinates. (01)
k) What is virtual displacement? (01)
l) Define : Degree of Freedom. (01)
m) Define variational principle. (01)
n) Write Newton's equation of motion in classical mechanics. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

(A) Evaluate: (07)

(i) $\int_0^\infty (x^c/c^x)\, dx; c > 1$

(ii) $\int_0^n x^n\, (n - x^p) dx$

(B) Prove that: (07)

(i) $\int_0^1 \frac{dx}{(1-x)^n} = \left(\frac{\sqrt{\pi}}{n}\right)\left(\frac{1}{n}\right)^{1/2}\left(\frac{1}{n}+\frac{1}{2}\right)^{-1/2}$

(ii) $\int_{-\infty}^{\infty} e^{-k^2x^2} = \sqrt{\pi}/k$

Q-3 **Attempt all questions** **(14)**

(A) Prove the following for the β function, where p > 0 and q > 0 ; (07)

(i) $\beta(p,q) = 2\int_0^{\pi/2} \sin^{2p-1}\theta.\cos^{2q-1}\theta.d\theta.$

(ii) $\beta(p,q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}\,dx.$

(B) Prove the following results for the γ function; where, $p > 0$, $a =$ Constant; $x = y$ (07)

(i) $\gamma(p) = \frac{1}{p}\int_0^{\infty} e^{-x^{\frac{1}{p}}}dx.$

(ii) $\gamma(p) = \int_0^1 \left[\log\left(\frac{1}{y}\right)\right]^{p-1} dy$

Q-4 **Attempt all questions** **(14)**

(A) Using Fourier series of function $f(x) = x^2$; $(-\pi < x < \pi)$, prove the following: (09)

(i) $\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right) = \frac{\pi^2}{6}$

(ii) $\sum_{n=1}^{\infty}\left[\frac{-1^n}{n^2}\right] = \frac{\pi^2}{12}$

and hence obtain value of

(iii) $\sum_{n=1}^{\infty}\left[\frac{1}{(2n-1)^2}\right]$

(B) Obtain the Fourier series to represent $f(x) = |x|$ in the interval $-\pi < x < \pi$ (05)

Q-5 **Attempt all questions** **(14)**

(A) Evaluate Fourier coefficients giving the general formula of Fourier series. (07)

(B) Explain the virtual work concept. Derive formula for De'Alembert's principle. (07)

Q-6 **Attempt all questions** **(14)**

(A) Derive Lagrange's Equation of Motion (09)

(B) Derive Hamilton's principle from Newton's equation. (05)

Q-7 **Attempt all questions** **(14)**

(A) Based on Lagrangian equation of motion, give the final formula for the Simple, Compound, Double and Spherical pendulums. Discuss and derive expression for any one of them. (08)

(B) Discuss and derive necessary formula for any one kind of L-C-R circuit using Lagrangian formulation and Rayleigh's dissipation function. (06)

Q-8 **Attempt all questions** **(14)**

(A) Obtain Hamilton's equation of motion and prove that Hamiltonian H is the total energy as the sum of kinetic energy and potential energy of the system. (07)

(B) Obtain Hamilton of simple pendulum with moving support. Also derive formula for the simple pendulum from Hamilton's equation. (07)